



Comparative study of some non-linear models for predicting the yield of *Gmelina arborea* plantation

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Abstract: Yield models are important for effective forest management and as such were developed for the University of Benin *Gmelina arborea* plantation, Nigeria. The objectives of the study were to develop, evaluate and compare predictions from some non-linear models for timber volume estimation. A total of nine non-linear models comprising of three models each for weibull, logistic and log-normal models were developed using the three independent variables combinations (Basal area and merchantable height, diameter at base and merchantable height, diameter at middle and merchantable height). The assessment criteria (correlation coefficient (R), coefficient of determination (R²), standard error of estimate (SE)) with the validation results (using percentage bias and probability plots of residuals) showed that all categories of weibull and logistic models generated in this study discovered to be very adequate for tree volume estimation. The highest R² (93.80), lowest SE (0.25) and lowest bias% (1.29) in the study were achieved from Weibull model 1a. The log-normal models were the least adequate for tree volume estimation with the highest bias%. The one way analysis of variance revealed that there were no significant differences in the performance of the non-linear models when varying predictor variables were used. The weibull, logistic models were therefore recommended for further use in this ecosystem and in any other forest ecosystem with similar site condition.

Keywords: Gmelina arborea, Plantation, Tectona grandis, Yield models

INTRODUCTION

Forest is long-lived dynamics biological systems that are continuously changing. It is often necessary to project these changes in order to obtain relevant information for sound decision making. In forestry, this information is acquired by means of forest inventories; systems for measuring the extent, quality and condition of forest (Husch et al., 2003). The main reason for conducting forest inventory either in the plantations or natural forest ecosystem is to estimate timber volume of the plots installed in the entire stand. For timber production, estimates of the growing stock are often expressed in terms of timber volume, which can be estimated from easily measurable dimensions of the tree. The most common procedure for volume estimation is to use volume equations, which are based on the relationship between volume and variables such as diameter, height etc. and other tree characteristics, which can be used in predicting tree volume. The assessment of stem volume is of high interest in forest management and is becoming of great global interest; for instance, in the context of Kyoto protocol rules, it is used in accounting for both absorbed and stored amount of CO₂ by trees (Lindner and Karjalainen, 2007). According to Avery and Burkhart (2002), volume equations are used to estimate average content of standing trees of various sizes and species. The reliability of volume estimates depends on the range and extent of the available sample data, and how well volume equations fit this sample data.

The University forest plantation was originally established with the sole aim of teaching and research purposes and improving the conservation and maintenance of the soil productivity, but it is now used as a production forest without neglect to the original objectives. It is therefore important that information on present yield is always available for effective management and planning. The cost of remeasurement of trees is high and hence the use of models for estimating volume becomes important. In the light of this it is important to update inventories, predict future yield and to explore management alternatives and silvicultural options in order to provide information for decision-making. In view of these, information on models for predicting yield will be valuable for a sustainable forest management decision-making. For example, Forest resources managers require tree volume information to produce yield estimates for timber inventory and improve forest management decision-making (Peng, 1999). The objective of this study was to develop, evaluate and compare some

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non-linear models for predicting the yield of University of Benin *Gmelina arborea* plantation.

MATERIALS AND METHODS

Study area: The G.arborea stand is located in the University of Benin Forest Plantation. However, this study covers an area of about 3.9 hectares of the forest plantation. The forest plantation is located between latitude 6.1°N to 6.8°N of the equator and 5.4°E to 6.0°E of the meridian (UNIBEN Master Plan, 1993). The Forest plantation is in the rainforest zone of Edo state, south-south, Nigeria. The rainforest is known for the bi-seasonal climate, which is the wet and dry season and it is generally moist because there is no prolonged period of drought. More importantly, precipitation usually exceeds or equals the potential evapotranspiration so that a shortage of soil moisture does not reasonably inhibit plant growth (Nwoboshi,1982). The university possesses an annual rainfall of 1500m. The humidity is also high and uniform ranging from 75% (afternoon) to 95% (morning). The topography of the area is gently slope eastward. The geology of the plantation consists of relatively thick layer of sedimentary sand and sandy clay of tertiary age (UNIBEN Master Plan, 1993). The sampling technique adopted for plot location is the systematic sampling. Seven sample plots were systematically selected from the G.arborea stand. The following tree data were collected in each sample plot: dbh (stem diameter at a position of 1.3 m above the ground level), diameters over bark at the base, middle and merchantable top, merchantable and total height. Data analysis

Basal area calculation: The basal area of all trees in the sample plots were calculated using the formula: BA = $(\prod D^2)/4$ 1

Where: BA= Basal area (m²), D= Diameter at breast height (cm), \prod = Pie (3.142)

Volume calculation: The volume of each tree was calculated in every plot using the Newton's formula:

$$\pi H \left[\frac{Db^2 + 4Dm^2 + Dt^2}{24} \right]$$

Where V = Stem volume (m³), H = Merchantable height (m), Db = Diameter at the base, Dm = Diameter at the middle, Dt = Diameter at the top and π = 3.142

Confidence limit (Upper and lower values): This is the range of values within which one might expect to find the parameter with same degree of assurance and was estimated as:

 $CL = Estimate (Volume/ha) \pm t\alpha/2(SE)$

Where, SE is the standard error of estimate

Analysis of variance (ANOVA): One way analysis of variance(ANOVA) was carried out to investigate the significant difference in the yield models using various predictor variables (Model using basal area and merchantable

height, model using diameter at the base and merchantable height, model using diameter at middle and merchantable height).

Non-Linear models

Weibull model: Ratkowsky (1983) and Myers (1986) employed weibull models in previous studies. The models is:

$$\omega(t) = \{ \alpha - \beta e^{-kt^{m}} \} + \varepsilon$$

The model was modified for this study as:

Logistic model:

Nelder (1961), Oliver (1964) The model was modified for this study as;

$$SV = \beta_0 / (1 + e^{\beta_1 X - \beta_2 MHT}) + \varepsilon$$

Log-normal model: The probability density function of a log-normal distribution is:

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$$SV = \frac{\beta_0}{BA\beta_1 \sqrt{2\pi}} e^{(InBA - MHT)}$$

Where SV=Stem Volume; MHT=Merchantable Height; X= Independent variable; β_0 , β_1 and β_2 = parameters.

Stem volume was the predicted variable. The predictor variables include merchantable height, basal area, diameter at the base and diameter at the middle. The merchantable height was used for all models in the study but the basal area was surrogated with diameter at the base and diameter at the middle. This resulted in 3 model types

Model 1: Weibull model:	
Model 1a: $SV = b_0 (1 - e^{-b1BA - b2MHT})$ 6	
Model 1b: $SV = b_0 (1 - e^{-b1Dbase - b2MHT})$	
Model 1c: $SV = b_0 (1 - e^{-b1Dmid - b2MHT})$	
Model 2: Logistic model:	
Model 2a: $SV = b_0/(e^{-b1BA-b2MHT})$	
Model 2b: $SV = b_0/(e^{-b1Dbase-b2MHT})$	
Model 2c: $SV = b_0/(e^{-b1Dmid-b2MHT})$ 11	
Model 3: Log-normal model	
Model 3a: $SV = 1/(BAb_0\sqrt{2\Pi})e^{(lnBA-b2MHT)}$	
(InDhase-b2MHT)	

Model 3b: $SV=1/(Dbaseb_0\sqrt{2}\Pi)e^{(lnDbase-b2MHT)}$13 Model 3c: $SV=b_0/(Dmidb_1\sqrt{2}\Pi)e^{(lnDmid-b2MHT)}$14 Where: b_0,b_1,b_2 = Regression constants to be estimated, E = The exponential constant (Euler's value = 2.71), BA= Basal area (m²), MHT= Merchantable Height, D_{base}=Diameter at the base, D_{mid}= Diameter at the middle, SV = Stem volume (m³).

All these models are intrinsically nonlinear regression models.

Assessment of the models: The volume models were assessed with the view of recommending those with good fit for further uses. The following statistical criteria will be used: Significance of regression (F-ratio), multiple correlation coefficient (R), coefficient of determination (R^2).

Validations of the models: This was achieved by comparing the models' output with values observed on the field. The validation process examines the usefulness or validity of the models (Marshall and Northway, 1993). The entire field data were divided into two sets. The first set (calibrating set) and the second the validation set. The calibrating set is used to construct the models while the validation set is used to test them. To test the accuracy of the yield models, the models were estimated with 20% of the plots randomly selected from the 20 plots available. The model was then used to predict the state of the remaining plots at the time of their current measurement, given their state at the previous inventory. For models with good fit, the intercept must be close to 0 and the slope close to 1, the model must be significant and highly correlated, coefficient of determination value must be very high and the standard of error of estimate must be small values (Onyekwelu and Akindele, 1995; Adekunle et al., 2004).

Percentage bias estimation: The absolute percentage difference (% bias) was determined by dividing the difference between volumes obtained with Newtons formula (observed volume) and models output by the same observed volume and multiplied by 100.

%Bias = (Vp-Vo)/Vo15

Where: V_o = The observed volume, V_p = The predicted volume (models output).

The value must be relatively small for the model to be acceptable for management purpose.

RESULTS AND DISCUSSION

Preliminary analysis: Descriptive statistics for the *G. arborea* stand (Table 1) showed that basal area ranged from $0.076m^2$ to $0.230m^2$, stem volume ranged from $0.820m^3$ to $3.600m^3$, merchantable height ranged from 6.57 to 17.07m and diameter at breast height ranged from 0.23m to 1.70m. The mean stem volume, mean basal area and mean merchantable height were 1.473m³, 0.106m² and 14.447m, respectively.

Assessment of models: All weibull models in the study (Table 2) were discovered to have good fit even when different independent variables (Basal area, diameter at the base and diameter at the middle) were used. This is as a result of the high correlation coefficient (R) and coefficient of determination (R^2) values, small standard error of estimate and significant

f-ratio (p<0.05) obtained. The R^2 and R values for the models in the *G. arborea* stand were 0.926 and 0.962 (model 1a), 0.856 and 0.925 (model 1b) and 0.811 and 0.900 (model 1c), respectively. The standard error (SE) ranged from 0.005 to 0.551.The logistic models in the study (Table 3) was discovered to also have good fit. The R^2 and R values recorded for the logistic models were 0.720 and 0.849 (model 2a), 0.700 and 0.837 (model 2b) and 0.598 and 0.774 (model 2c), respectively. The SE of estimate ranged from 0.001 to 2.096. However, the log-normal models (Table 4) had an R^2 and R values of 0.321 and 0.567(model 1a), 0.297 and 0.545 (model 3b) and 0.220 and 0.469 (model 3c), respectively. The SE ranged from 0.004 to 3.869.

Comparative assessment and validation of models types with varying predictive variables: The comparative assessment of the performance of the various model type (Basal area, diameter at base and diameter at middle) used in developing the non-linear equations (Weibull, logistic, log-normal) is showed in table 5. It was observed that there was no significant difference in the performance of yield prediction for three models combinations (Basal area and merchantable height, diameter at base and merchantable height, diameter at middle and merchantable height) for each of weibull, logistic and log-normal models at 5% level of significance. Generally, the highest value of percentage biases when the output of each model was compared with the observed volume was 28.262% (model 3b) across all the models (Table 6). For the weibull models, the range of the bias (%) was 1.290, 3.327 and 2.648 for model 1a, 1b and 1c, respectively. The logistic model had the values of bias (%) ranged from 1.426 (model 2c) to 3.870 (model 2b) while for the log-normal models, the percentage biases were 17.719 (3a), 28.262 (3b), 11.81(3c).

According to Adekunle (2007) non-linear models were discovered to be very adequate for yield estimation in lowland rainforest ecosystem. This is a further confirmation to the use of non-linear models in this study. The assessment criteria revealed that all weibull and logistic models were very suitable for tree volume estimation in the plantation except the log-normal models. These models were similar to those used by Yaoxiang et al. (2011) for Predict Stem Cumulative Biomass of Standing Trees and Yevide et al., (2014) for some plantation species also. In their studies, age was used as independent variable as Vanclay (1994) noted that there is no best method for forest growth models, this study therefore surrogated age with basal area, diameter at the base and middle. VanderSchaaf (2008) also replaced age with diameter and height during model generation in their studies. The indices of fit, which are the coefficient of multiple correlation and coefficient of determination (R and R^2 , respectively), for all models except the log-normal models were very high. During validation with simple linear regression equations (comparison of models outputs with

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Variables	Mean	Minimum	Maximum	SD	SE
THT	17.2691	9.4000	20.8200	3.9065	0.2215
MHT	14.4471	6.5700	17.0700	3.5056	0.1988
BA	0.1059	0.0760	0.2300	0.3761	0.0213
DB	0.6121	0.3300	1.1450	0.5033	0.0285
DM	0.2698	0.1700	0.5100	0.2399	0.0136
DT	0.1173	0.0774	0.3200	0.1726	0.0098
SV	1.4730	0.8200	3.6000	0.1827	0.0104
CD	6.2457	4.0873	12.2319	3.9044	0.2214
DBH	0.3194	0.2300	1.7000	0.2784	0.0158

THT- Total height; MHT- Merchantable height; BA- Basal area; DB- Diameter at base; DM-Diameter at middle; DT-Diameter at top; SV – Stem volume; CD- Crown diameter; DBH- Diameter at breast height; CL- Crown length; CR- Crown ratio; SLC- Slender coefficient

 Table 2. Weibull models for G.arborea stand.

S.N.	Models	Parameter	Estimates	SE	t-value	P-level
1a	$SV=b_0(1-e^{-b1BA-b2MHT})$	Во	0.853	0.056	43.023	0.002
	$R^2 = 0.926$, $R = 0.962$	b_1	2.509	0.502	13.045	0.000
		b_2	0.006	0.551	5.023	0.000
1b	$SV=b_0(1-e^{-b1Dbase-b2MHT})$	Во	0.225	0.017	52.455	0.000
	R ² =0.856, R=0.925	b_1	1.384	0.061	4.512	0.000
		b_2	-0.009	0.005	12.199	0.000
1c	$SV=b_0(1-e^{-b1Dmid-b2MHT})$	Во	0.090	0.456	63.824	0.000
	$R^2 = 0.811, R = 0.900$	b_1	2.417	0.078	5.240	0.000
		b_2	0.004	0.051	6.034	0.000

SV=Stem volume,BA= Basal area, MHT= Merchantable height,Dbase=Diameter at the base, Dmid= Diameter at the middle, SE= Standard Error of Estimates and b_0, b_1, b_2 =Parameter.

 Table 3. Logistic models for G.arborea stand.

S.N.	Models	Parameter	Estimates	SE	t-value	P-level
2a	$SV = b_0/(e^{-b1BA-b2MHT})$	Bo	0.631	2.096	79.023	0.000
	$R^2 = 0.720$, $R = 0.849$	b_1	4.435	1.245	6.240	0.000
		b_2	0.012	0.481	12.561	0.000
2b	$SV = b_0/(e^{-b1Dbase-b2MHT})$	Во	0.263	0.017	114.309	0.000
	$R^2 = 0.700$, $R = 0.837$	b_1	1.412	0.241	3.218	0.000
		b ₂	0.004	0.002	13.274	0.000
2c	$SV = b_0/(e^{-b1Dmid-b2MHT})$	Во	3.069	0.076	72.800	0.000
	$R^2 = 0.598$, $R = 0.774$	b_1	2.940	0.013	4.390	0.000
		b ₂	-0.022	0.001	12.450	0.000

SV=Stem volume,BA= Basal area, MHT= Merchantable height, Dbase=Diameter at the base, Dmid= Diameter at the middle, SE= Standard Error of Estimates and b_0, b_1, b_2 =Parameter.

S.N.	Models	Parameter	Estimates	SE	t-value	P-level
3a	$SV=b_0/(BAb_1\sqrt{2\prod})e^{(InBA-b2MHT)}$	Во	0.688	3.166	13.750	0.000
	R ² =0.321 , R=0.567	b_1	3.518	0.887	9.824	0.000
		b_2	0.011	0.605	4.093	0.000
3b	$SV=b_0/(Dbaseb_1\sqrt{2\prod})e^{(InDbase-b2MHT)}$	Во	0.168	3.092	62.300	0.000
	R ² =0.297, R=0.545	b_1	2.633	7.869	4.502	0.000
		b ₂	-0.017	1.094	11.095	0.000
3c	$SV=b_0/(Dmidb_1\sqrt{2\prod})e^{(InDmid-b2MHT)}$	Во	7.108	0.021	97.237	0.000
	$R^2 = 0.220, R = 0.469$	b_1	4.319	0.032	8.028	0.000
		b_2	-0.002	0.501	23.237	0.000

Table 4. Log-normal models for G. arborea stand.

 $SV=Stem volume, BA=Basal area, MHT=Merchantable height, Dbase=Diameter at the base, Dmid=Diameter at the middle, SE=Standard Error of Estimates and b_0, b_1, b_2 = Parameter.$

Table 5. ANOVA table showing the level of significant differences between the models types used in fitting the non-linear models.

Models		Sum of squares	df	Mean Square	F	Sig.
Weibull	Model Type	0.108	2	0.054	1.432	0.241ns
	Error	8.395	223	0.038		
	Total	8.503	225			
Logistic	Model Type	0.091	2	0.045	1.873	0.156ns
	Error	5.409	223	0.024		
	Total	5.500	225			
Log-normal	Model Type	0.031	2	0.016	0.775	0.462ns
	Error	4.460	223	0.020		
	Total	4.491	225			

 Table 6. Validation results and % bias of the Weibull models with simple linear regression model.

Models	b ₀	b ₁	R	\mathbf{R}^2 (%)	SE	Predicted volume	Bias (%)
1a	-0.067	4.356	0.969	93.80	0.254	1.492 ±0.14	1.290
1b	-0.043	2.765	0.866	74.98	0.318	1.424 ±0.23	3.327
1c	-0.075	1.095	0.855	73.04	0.630	1.434 ±0.37	2.648
2a	-0.058	3.656	0.908	82.46	0.254	1.441 ±0.34	2.172
2b	-0.136	6.344	0.900	81.09	0.318	1.416±0.31	3.870
2c	-0.092	2.455	0.837	70.03	0.630	1.452 ± 0.13	1.426
3a	-0.067	4.356	0.885	78.40	0.254	0.212 ±0.43	17.719
3b	-0.043	2.765	0.876	76.67	0.318	0.204 ± 0.39	28.262
3c	-0.075	1.095	0.715	51.06	0.630	0.299 ± 0.42	11.813

observed volume), the intercepts (b_o) were very close to zero while the slope coefficients were very close to 1. Also, the index of fit, R and R² values were high and significant F-ratio at P \leq 0.05 was obtained. The standard error of estimate is a good measure of overall predictive value of regression equations (Akindele and LeMay, 2006). It is also a common measure of

goodness of fit in nonlinear regression models (Glantz and Slinker, 2001), with low values indicating better fit. In view of the above, among the non-linear models, Weibull and logistic model gave the best volume predictions for *G.arborea* stands .When compared with observed volume, weibull model 1a (basal area and merchantable height) best predicted the yield. Other variable combination using weibull also gave close estimates of the yield prediction. This therefore means that diameter at the middle and other growth variables should be considered in further studies when developing models for yield prediction. Although the result of the one way analysis of variance showed that there was no significant difference (P>0.05) in fitting different independent variables in the non-linear models. This might be due to the fact that variation in diameter at various points (taper) on an individual tree is small as a result of the uniform spacing and even aged stands that characterized the forest plantation in the study. This would be revealed if further studies are carried out in a natural forest.

Conclusion

Non-linear models for volume estimation was developed and validated for G. arborea at the University of Benin forest plantation. The tree growth data were collected from sample plots in the study area. Based on the evaluation of the models examined in this study the weibull and logistic models were found to be more suitable and fit for volume prediction. However the best timber volume predictions was achieved from weibull model 1a. The log-normal models which gave the three highest bias% were the least adequate for yield estimation of the three models. Development of statistical models for growth and yield relationships of economic tree species is an essential preliminary step towards the implementation of sustainable forest management. The yield models developed in this study gave reasonable precise estimates of volume and is recommended for use in the management of the University of Benin Plantation and will serve as a framework for future data collection, management and analysis.

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